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Exploration of Triangle Similarity: Applying it to Derive the Spherical Mirror Formula

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Abstract

Similar triangles share identical shapes but may vary in size, whereas congruent figures are identical in both shape and size. This study focuses on spherical mirrors, which encompass two types: concave and convex. The spherical mirror formula serves as a vital equation, linking the object distance (the distance from the object to the mirror), the image distance (the distance of the formed image), and the focal length of the mirror. Analysis of ray diagrams reveals that light rays in spherical mirrors adhere to the concept of triangle similarity. When an object is positioned in front of a spherical mirror, light rays reflect, with corresponding angles of incidence and reflection found to be equal. This property enables the establishment of triangle similarity among the object, the mirror's surface, and the image. By capitalizing on the similarity of these triangles, proportions among the object distance, image distance, and focal length are established, leading to the derivation of the spherical mirror formula. This formula offers valuable insights into image formation and magnification within spherical mirrors, solidifying its significance as a fundamental concept in optics. In essence, this paper delves into the pivotal role of similar triangles in formulating the spherical mirror formula, providing deeper understanding into image formation and magnification in spherical mirrors.

Keyword: Similar, triangle, spherical, mirror

1. Introduction

Triangles having the same shape but different sizes are known as similar triangles [3]. Two congruent triangles are always similar, but similar triangles need not be congruent [9]. Two geometrical figures having exactly the same shape and size are said to be congruent figures. Two triangles are said to be congruent if the sides and angles of one triangle are exactly equal to the

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corresponding sides and angles of the other triangle [2].

A spherical mirror or a mirror that is a part of a sphere is a mirror that has the shape of a piece that is cut out of a spherical surface or material. There are two types of spherical mirrors: concave and convex a mirror that has the shape of a piece that is cut out of a spherical surface or material [4]. There are two types of spherical mirrors: concave and convex. The curved surface we see on a shining object or a shiny spoon can be considered a curved mirror [3]. The most widely and commonly used type of curved mirror is a spherical mirror. The reflecting surface of such mirrors is considered to be part of the surface of any sphere. Those mirrors that possess reflecting surfaces that are spherical are called spherical mirrors [4].

During the derivation of the spherical mirror formula, it is observed that the properties of similar triangles are used to derive the formula.

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Where,

Object distance is represented by u

Image distance is represented by v

Focal length of mirror is represented by f

2. Pre-melinieries

2.1. Definition of similar triangles

Similarity between two triangles is shown if

i) Their corresponding sides are proportional.

ii) Their corresponding angles are equal.

2.2. Properties of similar triangles

Triangles that are similar to one another have the following characteristics:

- Triangles that are similar to one another have the same basic shape but may vary in terms of size [13].

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- Each angle in a pair that corresponds to one another in any of the two identical triangles is the same.
- There is no difference in the ratio between any pair of matching sides of two different triangles.

2.3. Characteristic properties of similar triangles

- (AAA Similarity) Two triangles are said to be equivalent if they both have equal angles [13].
- (SSS Similarity) Triangles are similar if the ratios of their parallel sides are equal [13].
- (SAS Similarity) A similarity between two triangles is established if one set of parallel sides is proportionate to the other and the included angles are also equivalent [13].

2.4. Results based upon characteristic properties of similar triangles

- When comparing two triangles to determine whether or not they are equiangular, the ratio of their medians to the ratio of their sides should be equal [3, 9].
- The ratio of the sides that correspond to each other in two equiangular triangles is the same as the ratio of the segments that make up the angle bisectors of each triangle individually [3, 9].
- If two triangles are equiangular, then the ratio of the sides that correspond to their respective altitudes will be equal to the ratio of the heights that correspond to their sides.
- Similar triangles have an angle that is the same as an angle in another triangle, and their bisectors cut opposing sides in the same proportion [3, 9].
- If two of a triangle's sides and the median that cuts through the middle of one of those sides are proportionate to the same two sides and the median that cuts through the middle of another triangle, then the two triangles are comparable to one another [3, 9].

- If the ratios of the median, two sides, and third side of one triangle are the same as those of another triangle, then we say that two triangles are identical [3, 9].

2.5. Theorem

When a line is drawn parallel to one side of a triangle and then cuts across the other two sides at right angles, the resulting division of those two sides is also right.

3. Application of properties of similar triangle in mirror formula

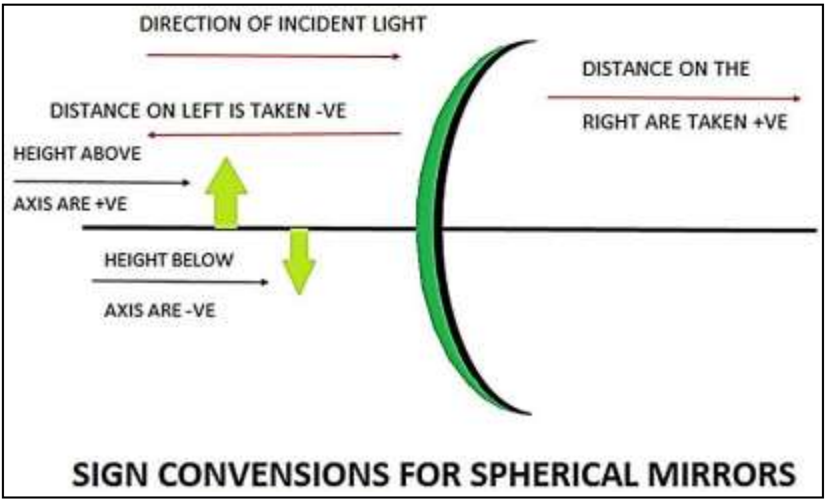

Fig 1: Spherical mirrors showing sign convention [4]

Mirror formula, often known as the spherical mirror formula, is among the most widely used formulae in optics. Mirror formula is defined as the distance among object's distance, image's distance and mirror's focal length. Plain and spherical mirrors, both convex and concave, may be calculated using the formula. The formula for a reflection is as follows.

The assumptions behind how the mirror formula is derived:

The mirror formula is obtained by taking into account the following

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factors:

- All of the image and object distances are being calculated relative to the mirror's pole [4].
- According to the rule governing sign conventions, positive signs indicate distances that are measured in the same direction as the incident ray, while negative signs indicate distances that are measured in the opposite direction [4].
- It is conventionally understood that distances below the axis are negative and those above the axis are positive [4].

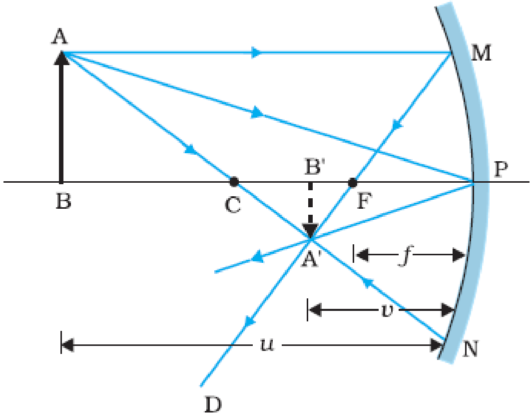

Fig 2: Ray diagram for to get an image from a set of spherical mirrors [4]

The following rules in the ray diagram are used to get an image from a set of spherical mirrors.

i) The line drawn from the point that runs perpendicular to the main axis. The beam of light that is reflected travels along the mirror's focal point.

ii) The beam that goes through focal point of concave mirror, or gives the impression of going through the focal point of a convex mirror. Rays that are reflected simply return along the same route they took to reflect.

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iii) The beam that travels through a convex mirror and seems to go through a concave mirror's focus. It's easy to see that the reflected beam lines up with the main axis.

iv) Any polar ray that enters at an angle. The reflected light complies the rules of reflection.

The ray diagram for three rays is shown in the figure. This diagram illustrates how a concave mirror produces a true image of an object AB, denoted by the notation A′B′.

We can now calculate the mirror equation, which describes the relationship between image distance (v), object distance (u), and focal length (f).

Similarities between the triangles A′B′F and MPF may be seen in the previous illustration. (In the case of paraxial rays, MP is equivalent to a straight line perpendicular to CP.) Hence,

$$\frac{B'A'}{PM} = \frac{B'F}{FP} \quad or \quad \frac{B'A'}{BA} = \frac{B'F}{FP} \; (because\ PM = AB) \qquad \dots (1)$$

Since < APB = < A′PB′, the right angled triangles A′B′P and ABP are also similar. Thus,

$$\frac{B'A'}{BA} = \frac{B'P}{BP} \qquad \dots (2)$$

Equations (1) as well as (2) are compared, and we obtain

$$\frac{B'F}{FB} = \frac{B'P - FP}{FP} = \frac{B'P}{BP} \qquad \dots (3)$$

In the third equation, we see a correlation between distance magnitudes. The convention of signs is currently being used.

In this case, we see that the object's light is reflected in the mirror MPN. Therefore, we see this as a step in the right direction.

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To go from pole P to object AB, image A′B′, and focus F, we have to move clockwise with respect to the direction of incident light. Therefore, we get negative signals for all three.

Therefore, BP = –u, FP = –f, B′ P = –v,

In Eq. (3) utilizing these, we obtain

$$\frac{-v+f}{-f} = \frac{-v}{-u} \quad or \quad \frac{v-f}{f} = \frac{v}{u} \quad or \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

The above equation is referred to as a mirror formula.

„ Same triangular concept is employed in the above formula deduction.

Various computer science principles and techniques can be implemented to improve understanding, analysis and practical applications. Here are some aspects of computer science that can be integrated into optics research and analysis:

I. **Numerical methods:** Computer science provides numerical methods and algorithms for solving complex equations. These methods can be used to solve for unknown variables in optical systems, making it easier to handle complex setups and nonlinear relationships.

II. **Simulation:** Computer simulations can model the behavior of light in various optical systems. This is especially valuable for complex scenarios where analytical solutions are challenging. Tools like ray tracing software can simulate how light rays travel through lenses, mirrors, and other optical components.

III. **Data analysis:** When working with experimental data in optics, computational techniques for data analysis and visualization (e.g., Python libraries such as NumPy, SciPy, and Matplotlib) can help extract meaningful insights from measurements, such as determining lens precision or analyzing aberrations.

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IV. **Optimization algorithms:** Optimization algorithms, often used in computer science, can be applied to lens design. They can help optimize lens parameters to achieve specific optical goals, such as minimizing aberrations or maximizing light transmission.

V. **Machine learning:** While not directly applicable to the lens formula itself, machine learning can be used in optical applications for tasks such as image recognition, quality control, or even automating the tuning of optical systems for optimal performance.

VI. **Image processing:** Computer vision techniques are widely used in optics, especially in image processing and analysis. This includes tasks such as image restoration, pattern recognition, and feature extraction that have applications in fields such as astronomy and microscopy.

VII. **Computer-Aided Design (CAD):** CAD software is widely used in the design of optical systems. Engineers and scientists can use CAD tools to create and optimize lens designs to ensure they meet specific requirements and constraints.

VIII. **Hardware and Control:** For experimental setups involving optics, computer science plays a role in controlling hardware components such as cameras, lasers, and detectors. Programming languages and protocols are used to interface with these devices and collect data.

IX. **Algorithm development:** The principles of computer science are fundamental to the development of algorithms that can effectively analyze optical data. For example, algorithms for image registration or phase retrieval in microscopy are heavily dependent on computational techniques.

X. **Data storage and management:** The storage and management of large data sets, especially in fields such as astronomy and high-resolution imaging, requires expertise in data storage technologies and databases.

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4. Conclusion

In summary, we have explored the profound link between the concept of similar triangles and the derivation of the spherical mirror formula. By recognizing the triangle similarity among the object, the mirror, and the image, we have derived a fundamental equation that correlates object distance, image distance, and focal length in spherical mirrors. This insight holds significant importance in the realm of optics. The application of similar triangles extends beyond the derivation of the formula; it serves as a valuable tool for determining the length or height of objects or images in optical scenarios. This understanding is particularly beneficial for secondary school students, as it illuminates the practical utility of the concepts they learn in geometry classes. Understanding the connection between similar triangles and the mirror formula allows students to develop a deeper appreciation for the elegance of mathematics and its real-world implications. Moreover, this knowledge empowers them to tackle more complex optical problems and enhances their analytical skills.

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