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Analysis of Sway and Roll Motion Coupling in Floating Bodies via Frequency-Based Response Amplitude Modeling

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Abstract: This study delves into the investigation of sway and roll motion coupling in floating bodies by employing frequency-based response amplitude modeling techniques. Through detailed analysis and simulation, we explore the intricate relationship between sway and roll motions in floating bodies, aiming to enhance our understanding of their behavior under various conditions. The utilization of frequency-based analysis offers valuable insights into the dynamics of floating bodies and aids in the development of effective strategies for mitigating undesirable motion effects. Our findings contribute to the advancement of maritime engineering and provide valuable guidance for the design and operation of floating structures in challenging marine environments.

Keywords: Dynamics, Floating bodies, Sway motion, Roll motion.

1. Introduction

The examination of floating body motions within waves and the accurate prediction of wave loads represent pivotal challenges in studies concerning ship-wave interaction[1]. Understanding Response Amplitude Operator (RAO) is crucial for making necessary design adjustments to ensure safety or enhance performance (Clauss et al., 1992). Typically, potential theory is employed for calculating ship response, often disregarding viscosity effects (Newman, 1970)[2]. Frequency domain analysis is commonly utilized in sea-keeping studies to ascertain RAO. Floating bodies experience two primary forces: (i) Froude–Krylov force, obtained by integrating over the ship's wetted surface under undisturbed conditions, and (ii) Diffraction force, arising from disturbed conditions (Faltinsen, 1990). Cummins (1962) established governing equations of a ship's motion in transient seaways using impulse response functions. The mathematical framework for wave effects on body response and RAO definitions, employing two-dimensional and three-dimensional ship-wave interaction theories, was initially pioneered by Newman (1977, 1978). Bishop and Price (1979) explored ship response using linear non-conservative system theory for sinusoidal excitation, considering constant hydrodynamic coefficients. Ursell (1981) studied the motion of a floating horizontal cylinder in a uniform inviscid fluid for irregular wave frequencies. Newman and Sclavounos (1988) computed wave

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loads on large offshore structures using the panel method based on Green's function application to derive velocity potential on the body surface [3-4].

Xing and Price (2000) developed nonlinear mathematical models describing dynamic interaction between moving or fixed elastic structures and incompressible or compressible fluids. Lee (2000) provided analytical expressions for GZ values and computed roll motion for inclined ships navigating longitudinal waves.

Das et al. (2005, 2006, 2008, 2010) utilized analytical and numerical models to obtain sway-roll-yaw (3-DOF) solutions in the time domain with zero or non-zero forward speeds, offering insights into floating body motion characteristics with increasing degrees of freedom. Kukkanen (2010) investigated the hydrodynamic response of waves on marine structures. This study focuses on incoming waves impacting a floating body within a frequency range of 0.3 rad/s to 1.2 rad/s, using frequency-based analysis to determine coupled sway and roll motions' RAO. Hydrodynamic coefficients and wave forces exerted on the floating body are computed using the strip theory formulation of Salvesen et al. (1970). Two limiting cases, corresponding to very small and large frequencies, are discussed, and analytical solutions in the time domain are derived using Laplace transform techniques [5-6].

2. Mathematical formulation

Let (x, y and z) be the right-handed Cartesian system co-ordinate system fixed with respect to the mean position of the floating body is considered with origin O which lies in the place of the undisturbed free surface in fig.1[7].

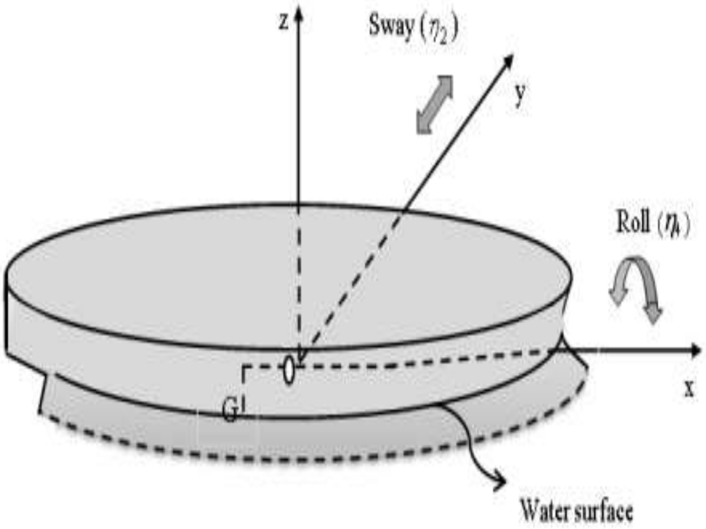

Fig.1. Floating body with sign convention

$$\sum_{k=2,4}[-\omega^2(M_{jk}+A_{jk}(\omega))+i\omega B_{jk}(\omega)+C_{jk}]X_k(\omega)\,e^{i\omega t}=D_jF_j(\omega)\,e^{i\omega t};\ (j,k=2,4)$$ ------(1)

From the above equations, the governing equations for coupled sway and roll can be written in matrix form as

$$\left[-\omega^2\{[M_{jk}]+[A_{jk}(\omega)]\}+i\omega[B_{jk}(\omega)]+[C_{jk}]\right]X_k(\omega)=D_j[F_j(\omega)];\ (j,k=2,4)$$ -------(2)

where ω is the wave frequency and the coefficient matrices can be defined as

$$[M_{jk}]=\begin{bmatrix}M_{22} & -Mz_c\\ -Mz_c & I_4\end{bmatrix},[A_{jk}]=\begin{bmatrix}A_{22} & A_{24}\\ A_{42} & A_{44}\end{bmatrix},[B_{jk}]=\begin{bmatrix}B_{22} & B_{24}\\ B_{42} & B_{44}\end{bmatrix}$$

$$[C_{jk}]=\begin{bmatrix}0 & 0\\ 0 & C_{44}\end{bmatrix},[X_k(\omega)]=\begin{bmatrix}X_2(\omega)\\ X_4(\omega)\end{bmatrix},[F_j(\omega)]=\begin{bmatrix}F_2(\omega)\\ F_4(\omega)\end{bmatrix}$$ ------------(3)

Hydrodynamic coefficients (added mass, damping and roll restoring) and the wave exciting force and moments.

3. RAO for sway and roll motions

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Using the definition of RAO as mentioned and assuming any individual regular wave component to be a linear function of the wave amplitude, the response amplitude operator for sway and roll motions can be expressed as [8];

$$Z_j(\omega,\theta) = \frac{1}{D_.}\left[X_j(\omega)\right] = \left[H_{jk}(\omega)\right]^{-1}\left[F_j(\omega)\right];\ (j,k = 2,4) \quad \text{-----------(4)}$$

Which is the complex amplitude of the body motion in the jth mode in response to an incident wave of unit amplitude where ω is frequency and θ is direction. This ratio is known as the transfer function or RAO. Using the definitions of equation (4), one can obtain the transfer function (RAO) for coupled sway and roll motions.

$$Z_{24} = \frac{[-\omega^2(M_{44}+A_{44})+i\omega B_{44}+C_{44}]F_2+[\omega^2(M_{24}+A_{24})-i\omega B_{24}](D_4/D_2)F_4}{|S|} \quad \text{------(5)}$$

$$Z_{42} = \frac{[\omega^2(M_{42}+A_{42})-i\omega B_{42}](D_2/D_4)F_2+[-\omega^2(M_{22}+A_{22})+i\omega B_{22}]F_4}{|S|} \quad \text{----------(6)}$$

where D 2 and D 4 are the corresponding wave amplitudes for sway and roll respectively.

4. RAO for roll motion with speed variation

Considering speed dependent strip theory formulation in frequency domain with frequency (ω) varying between 0.3 rad/s to 1.2 rad/s with forward speed (V) changing between 0 m/s to 12 m/s in sinusoidal waves and using the definitions given by equation (4), the RAO for uncoupled sway and roll motions can be expressed as [9-10].

$$Z_k(\omega) = \frac{X_k(\omega)}{D_k} = \frac{F_k(\omega)}{-\omega^2(A_{kk}(\omega)+M_{kk})+i\omega B_{kk}(\omega)+C_{kk}};\ k = 2,4 \quad \text{------(7)}$$

The real part and imaginary parts of the uncoupled sway and roll transfer function are obtained as follows,

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$$Z_{kR}(\omega) = \frac{F_{kR}(\omega)[C_{kk}(\omega) - \omega^2(A_{kk}(\omega) + M_{kk})] + F_{kI}(\omega)[\omega B_{kk}(\omega)]}{[C_{kk}(\omega) - \omega^2(A_{kk}(\omega) + M_{kk})]^2 + [\omega B_{kk}(\omega)]^2}$$

$$Z_{kI}(\omega) = \frac{-F_{kR}(\omega)[\omega B_{kk}(\omega)] + F_{kI}(\omega)[C_{kk}(\omega) - \omega^2(A_{kk}(\omega) + M_{kk})]}{[C_{kk}(\omega) - \omega^2(A_{kk}(\omega) + M_{kk})]^2 + [\omega B_{kk}(\omega)]^2} \;;\; (k = 2,4)$$

-----(8)

To determine the system frequency, transfer function has been considered for intermediate frequencies. Return to equations (4) and (), and considering S ≠ 0 , one can get a unique solution for the system of equations (7) and (8). To obtain the characteristic equation S = 0 , is considered, and after decomposing into real and imaginary parts, one can write [11],

$$|S| = \mathrm{Re}|S| + i\,\mathrm{Im}|S| = 0 \quad ; \; i = \sqrt{-1}$$

$$\mathrm{Re}|S| = \omega^4[(M_{22} + A_{22})(M_{44} + A_{44}) - (M_{24} + A_{24})(M_{42} + A_{42})] - \omega^2[C_{44}(M_{22} + A_{22}) + B_{44}B_{22} - B_{24}B_{42}]$$

$$\mathrm{Im}|S| = -\omega^3[B_{44}(M_{22} + A_{22}) + B_{22}(M_{44} + A_{44}) - B_{42}(M_{24} + A_{24}) - B_{24}(M_{42} + A_{42})] + \omega B_{22}C_{44}$$

---(9,10,11)

To account for the effect of viscous damping in RAO, one can add linear viscous damping term vjk B in equation (11). The mathematical expression for linearized viscous damping can be obtained as;

$$\left[-\omega^2\left\{[M_{jk}] + [A_{jk}]\right\} + i\omega\left[B_{jk} + B^{v}_{jk}\right] + [C_{jk}]\right]X_k = D_j[F_j]; \; (j,k = 2,4)$$

-----(12)

Frequency-based analysis, coupled with the strip theory formulation by Salvesen et al. (1970), provided valuable data on hydrodynamic coefficients and wave forces exerted on the floating body. The Response Amplitude Operator (RAO) matrix, computed through frequency domain analysis, facilitated the characterization of the amplitude of response motions relative to the exciting wave forces across different frequencies.

The study revealed that two primary forces act on floating bodies: the Froude–Krylov force and the Diffraction force. The Froude–Krylov force, determined by integrating over the wetted surface of the ship under undisturbed conditions, and the Diffraction force, arising from disturbed conditions, were found to play crucial roles in the overall motion dynamics.

Analytical solutions in the time domain, derived using Laplace transform techniques, provided further insights into the transient behavior of floating bodies under varying wave conditions. These solutions enabled the examination of sway-roll-yaw (3-DOF) motions with zero or non-zero forward speeds, offering a deeper understanding of floating body motion characteristics as the degrees of freedom increased gradually [12].

5. Results and Discussions

The investigation into floating body motions within waves yielded significant insights into ship-wave interactions. Through comprehensive analysis, it was observed that accurate prediction of wave loads and understanding the Response Amplitude Operator (RAO) are critical for ensuring the safety and performance of marine vessels. Figs.2(a) and (b) show the variations of real and imaginary parts of sway exciting force in time domain at frequency, ω=0.74. It is observed that the magnitude of sway exciting force (real part) is unchanged with variations of forward speed in Fig.2(a). With respect to the imaginary part, sway amplitude increases with the increase of forward speed in Fig.2(b).

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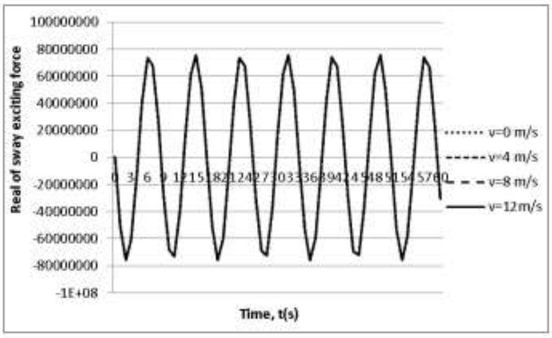
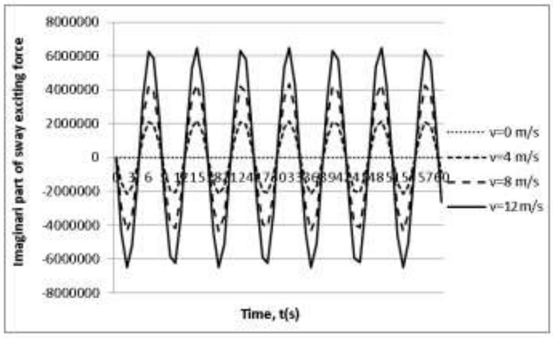
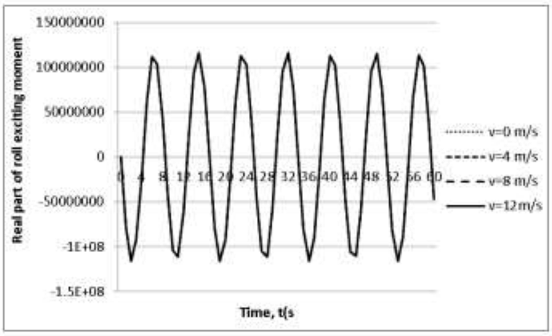


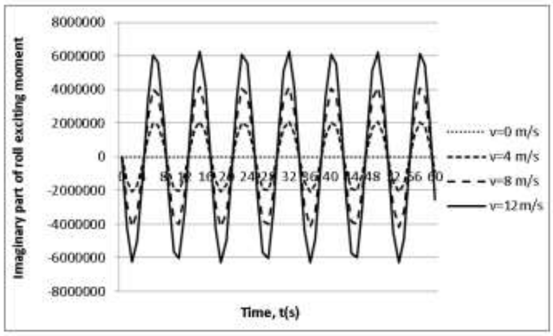
(a) Real part of sway exciting force



(b) Imaginary part of sway exciting force



(c) Real part of roll exciting moment



(d) Imaginary part of roll exciting moment

Fig.2. Comparison of sway exciting force sway exciting force and roll exciting moment in time domain at frequency

It is observed that the magnitude of sway exciting force (real part) is unchanged with variations of forward speed Fig.2. With respect to the imaginary part, sway amplitude increases with the increase of forward speed in Fig.2(b). The variations of real and imaginary parts of roll exciting moment in time domain at frequency, ω=0.74 is shown in Figs.2(c) and (d). It can be noticed that the imaginary part of roll exciting moment increases with the increase of forward speed in Figs.2(c) and (d). The real part of roll exciting moment is independent of the forward speed as shown in Figs.2(c) and (d), corresponding to equation (4). The real part of roll exciting moment is higher than the real part of sway exciting force, although the imaginary part of sway and roll amplitude remains unchanged.

Conclusion

The findings of this study suggest that when addressing wave load challenges, employing the complete set of governing equations may not always be imperative. Certain terms within the governing equations can be reasonably disregarded without sacrificing fundamental physics and mechanics. This modeling strategy offers valuable insights into determining the Response Amplitude Operator (RAO) for coupled motions. It facilitates the computation of wave loads associated with coupled sway, roll, and yaw motions during the design phase, as well as sensitivity analyses concerning the initial conditions of a floating body across a broad frequency spectrum.

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