



Exploring Mathematical Modeling in Fluid Flow Applications Through Partial Differential Equations

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Abstract

The core concept of the Functioning of Derivates approach lies in organizing optimization problems into multiple stages, which are systematically solved one stage at a time. While each stage involves solving an ordinary optimization problem, its outcome informs the characteristics of the subsequent stage in the sequence. Often, these stages represent distinct time periods within the problem's planning horizon. For instance, the task of determining inventory levels for a single commodity can be formulated as a derivate program. In practical applications, optimization variables may not always be continuous. In some instances, all or some variables must be chosen from a set of integer or discrete values.

Keywords: *derivates, mathematical functions*

1. INTRODUCTION

The famous naturalist Charles Darwin defined natural selection or survival of the fittest in his book (Darwin, 1929) as the preservation of favourable individual differences and variations, and the destruction of those that are injurious. In nature, individuals have to adapt to their environment in order to survive in a process called evolution, in which those features that make an individual more suitable to compete are preserved when it reproduces, and those features that make it weaker are eliminated. Such features are controlled by units called genes, which form sets known as chromosomes. Over subsequent generations not only the fittest individuals survive, but also their genes which are transmitted to their descendants during the sexual recombination process, which is called crossover. A review of the methods for discrete variable optimization was recently presented by Bremicker et al. (1990), Vanderplaats and Thanedar (1991) and Arora et al. (1994). Several algorithms for discrete optimization problems were developed, among them branch and bound method, penalty function approach, rounding-off, cutting plane, simulated annealing, genetic algorithms, neural networks, and Lagrangian relaxation methods. It is observed that some of the methods for discrete variable optimization use the structure of the problem to speed up the search for the discrete solution. This class of methods is not suitable for implementation into a general purpose application (Arora et al., 1994). The branch and bound method, simulated annealing, and genetic algorithm are the most used methods. Herein, the literature review will be focused on these methods in the following sections.



2. BACKGROUND OF THE STUDY

This paper, published in two parts, is mainly concerned with general properties of Dini derivatives of functions of one and several variables and with some applications of this topic to the study of generalized convexity and generalized optimality conditions for mathematical programming problems. In part I the basic definitions and properties are given, with reference both to functions of one real variable and to functions of several real variables. In this part special attention is given to the restatement of the basic theorems of the classical analysis to non-differentiable functions, in terms of Dini derivatives. In part II we use these derivatives in order to define some classes of non-differentiable generalized convex functions and the class of generalized upper quasi-differentiable functions. This part concludes with the development of optimality conditions for a non-smooth programming problem, expressed in terms of the tools previously introduced.

3. SCOPE OF WORK

This article aims to provide a process that can be used in financial risk management by resolving problems of minimizing the risk measure (VaR) using derivatives products, bonds and options. This optimization problem was formulated in the hedging situation of a portfolio formed by an active and a put option on this active, respectively a bond and an option on this bond. In the first optimization problem we will obtain the coverage ratio of the optimal price for the excretion of the option which is in fact the relative cost of the option's value. In the second optimization problem we obtained optimal exercise price for a put option which is to support a bond.

4. OBJECTIVES

1. To check the developments in derivatives of mathematics with genetic algorithm.
2. To find out the scope and need of derivatives in mathematics with applications.
3. To find the Applications of the Derivative.
4. To check the Applications of Derivatives to Business and Economics.
5. To check A Collection of Problems in Differential Calculus.
6. To find An Optimization of the Risk Management using Derivatives.

5. LITERATURE SURVEY

Hobden P (2006) this article aims to provide a process that can be used in financial risk management by resolving problems of minimizing the risk measure (VaR) using derivatives products, bonds and options. This optimization problem was formulated in the hedging situation of a portfolio formed by an active and a put option on this active, respectively a bond and an option on this bond. In the first optimization problem we will obtain the coverage ratio of the optimal price for the excretion of the option which is in fact the relative cost of the option value. In the second optimization problem we obtained optimal exercise price for a put option which is to support a bond.



Maharajh N, Brijlall D & Govender N (2008) In the article's first part we obtained the coverage ratio of the optimal exercise price of options which is the relative cost at the option's value by an optimization problem of VaR in the situation of a portfolio consisting in a financial asset and a put option. In the second part of this article we obtained the optimal exertion price of a put option in a portfolio in which is a bond and this option. And for this result it has been created a minimum problem of VaR and the obtained result leads to the idea that VaR linear depends of the h percent of the bond cover by options.

Marrongele K(2010) Many important applied problems involve finding the best way to accomplish some task. Often this involves finding the maximum or minimum value of some function: the minimum time to make a certain journey, the minimum cost for doing a task, the maximum power that can be generated by a device, and so on. Many of these problems can be solved by finding the appropriate function and then using techniques of calculus to find the maximum or the minimum value required.

Bezuidenhout J (2001) This qualitative case study in a rural school in Umgungundlovu District in KwaZulu-Natal, South Africa, explored Grade 12 learners' mental constructions of mathematical knowledge during engagement with optimisation problems. Ten Grade 12 learners who do pure Mathematics participated, and data were collected through structured activity sheets and semi-structured interviews. Structured activity sheets with three tasks were given to learners; these tasks were done in groups, and the group leaders were interviewed. It was found that learners tended to do well with routine-type questions, implying that they were functioning at an action level. From the interviews it appeared that learners might have the correct answer, but lacked conceptual understanding.

6. METHODOLOGY

These methods approximate an unknown function f by a response surface (or meta model) \hat{f} . Any mismatch between f and \hat{f} is assumed to be caused by model error and not because of noise in experimental measurements. Response surfaces may be non-interpolating or interpolating. The former are obtained by minimizing the sum of square deviations between f and \hat{f} at a number of points, where measurements of f have been obtained. The latter produce functions that pass through the sampled responses. x

$$\hat{f}(x) = \sum_{i=1}^m \alpha_i f_i(x) + \sum_{i=1}^p \beta_i \varphi(x - x^{(i)})$$

Where f_i are polynomial functions, α_i and β_i are unknown coefficients to be estimated, φ is a basis function, and $x^{(i)} \in \mathbb{R}^n$, $i=1, \dots, p$, are sample points. Basis functions include linear, cubic, thin plate splines, multi quadratic, and kriging. These are discussed below in more detail. Kriging Originally used for mining exploration models, kriging models a deterministic response as the realization of a stochastic process by means of a kriging basis function.

$$\hat{f}(x) = \mu + \sum_{i=1}^p b_i \exp \left[- \sum_{h=1}^n \theta_h |x_h - x_h^{(i)}|^{p_h} \right],$$

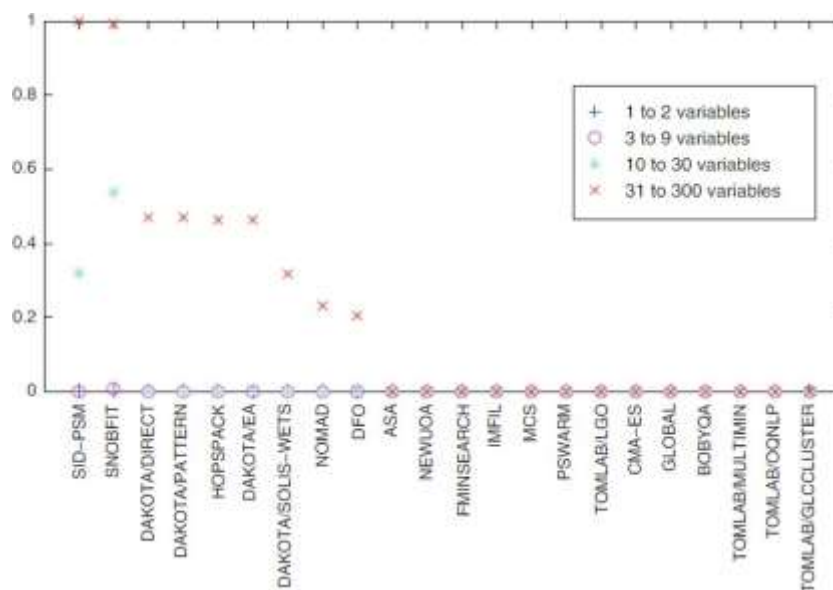
$\theta_h \geq 0, \quad p_h \in [0, 2], \quad h = 1, \dots, n.$

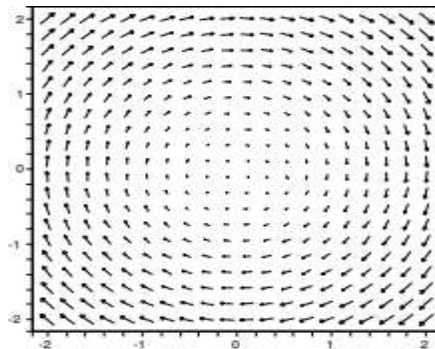


Assuming \hat{f} is a random variable with known realizations $\hat{f}(x(i)), i=1, \dots, p$, the parameters μ , b_i , θ_h and ϕ_h are estimated by maximizing the likelihood of the observed realizations. The parameters are dependent on sample point information but independent of the candidate point x . Nearby points are assumed to have highly correlated function values, thus generating a continuous interpolating model. The weights θ_h and ϕ_h account for the importance and the smoothness of the corresponding variables. The predictor $\hat{f}(x)$ is the minimized over the entire domain.

7. RESULTS

In order to assess the quality of the solutions obtained by different solvers, we compared the solutions returned by the solvers against the globally optimal solution for each problem. A solver was considered to have successfully solved a problem during a run if it returned a solution with an objective function value within 1% or 0.01 of the global optimum, whichever was larger. In other words, a solver was considered successful if it reported a solution y such that $f(y) \leq \max(1.01 f(x^*), f(x^*) + 0.01)$, where x^* is a globally optimal solution for the problem. To obtain global optimal solutions for the test problems, we used the general-purpose global optimization solver BARON to solve as many of the test problems as possible. Unlike derivative-free solvers, BARON requires explicit algebraic expressions rather than function values alone. BARON's branch-and-bound strategy was able to guarantee global optimality for most of the test problems, although this solver does not accept trigonometric and some other nonlinear functions. For the latter problems, LINDOGLOBAL was used to obtain a global solution.

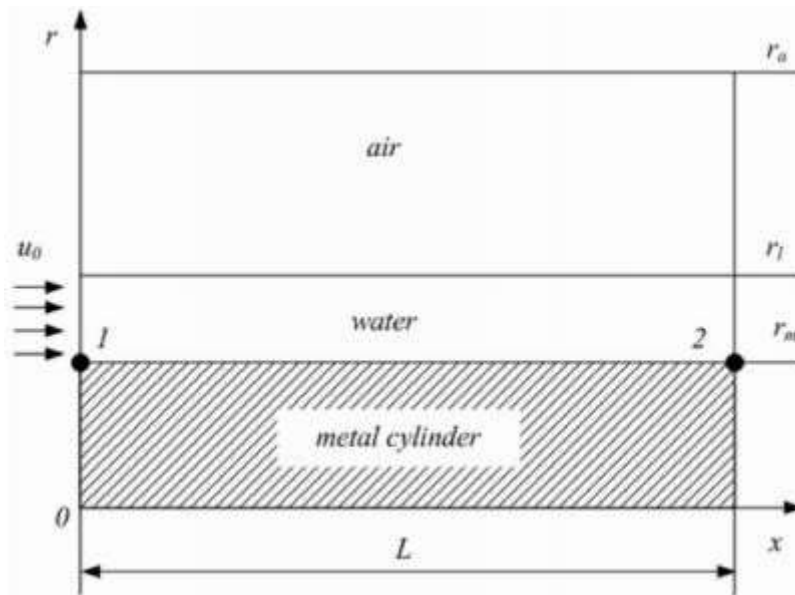




Examples of divergence free vector fields in R^2

Most instances were solved within a few minutes. Since the test problems are algebraically and computationally simple and small, the total time required for function evaluations for all runs was negligible. Most of the CPU time was spent by the solvers on processing function values and determining the sequence of iterates. A limit of 10 CPU minutes was imposed on each run.

A high-temperature solid metal cylinder of radius r_m and length L is cooled with a water flow moving longitudinally in the direction of the axis u and having the initial rate $0 < u < u_0$ and temperature T_{01} . The water thickness is l in r_1 - r_m . Above the water layer there is a layer of air with thickness al in r_1 - r_m . The physical diagram of the calculation region is presented



The physical diagram of the calculation region

The system of equations for the gas-liquid medium

$R_m < R < R_a$

$$\bar{\rho} \frac{\partial u}{\partial t} + \bar{\rho} u \frac{\partial u}{\partial x} + \bar{\rho} v \frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \bar{\mu} \frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} r \bar{\mu} \frac{\partial u}{\partial r} \dots\dots\dots (1)$$

$$\bar{\rho} \frac{\partial v}{\partial t} + \bar{\rho} u \frac{\partial v}{\partial x} + \bar{\rho} v \frac{\partial v}{\partial r} = -\frac{\partial p}{\partial r} + \frac{\partial}{\partial x} \bar{\mu} \frac{\partial v}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} r \bar{\mu} \frac{\partial v}{\partial r} - \bar{\mu} \frac{v}{r^2} \dots\dots\dots (2)$$



$$\frac{\partial \bar{p}}{\partial t} + \frac{\partial(\bar{p}u)}{\partial x} + \frac{1}{r} \frac{\partial(r\bar{p}v)}{\partial r} = 0 \quad \dots\dots\dots (3)$$

$$\frac{\bar{p}}{\rho c} \frac{\partial T}{\partial t} + \bar{p}cu \frac{\partial(\bar{p}u)}{\partial x} + \bar{p}cv \frac{\partial T}{\partial r} = \frac{\partial}{\partial x} \bar{\lambda} \frac{\partial T}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} r\bar{\lambda} \frac{\partial T}{\partial r} - \dot{m}Q \quad \dots\dots\dots (4)$$

$$\bar{p} \frac{\partial Y}{\partial t} + \bar{p}u \frac{\partial Y}{\partial x} + \bar{p}v \frac{\partial Y}{\partial r} = \dot{m} \quad \dots\dots\dots (5)$$

The specific mass rate of the vapor formation is found from the heat balance equation

$$\dot{m} = \left(\bar{p}c \Delta T^* \right) / Q,$$

where the reduced heat flow is defined according to the following equation

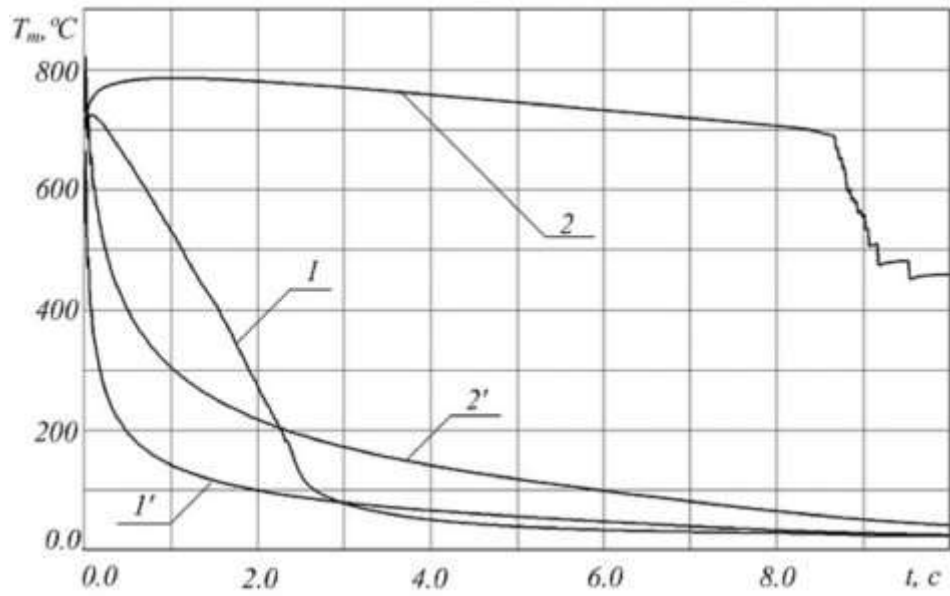
$$\Delta T^* = \begin{cases} 0, & \text{if } T(t+\Delta t) < T_s \\ [T(t+\Delta t) - T(t)] / \Delta t, & \text{if } T(t+\Delta t) > T_s \end{cases}$$

The energy equation for the metal cylinder

$$0 < r < r_m$$

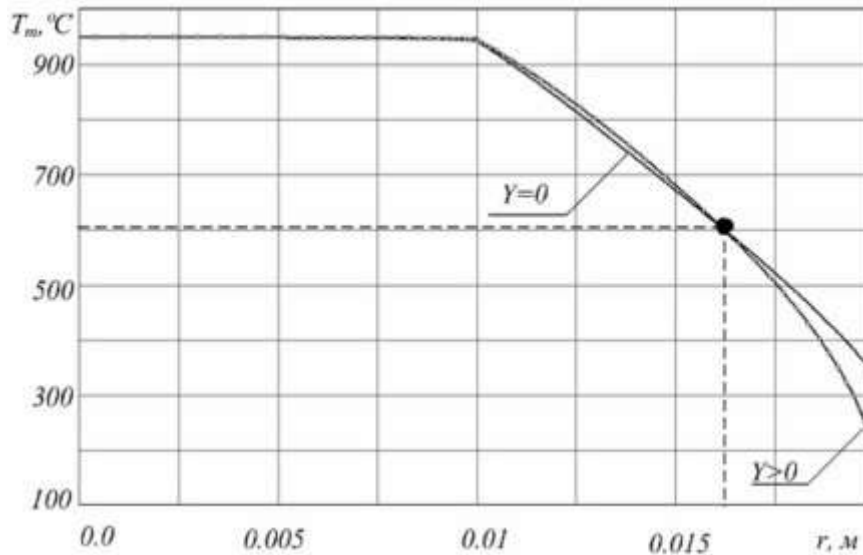
8. Calculation results:

The results obtained with the use of the offered mathematical model allowing solving the conjugate problem of convective heat exchange taking into account the vapor formation with the results calculated by the mathematical model using criterial equations for determining the boundary conditions on the cylinder surface at the contact with the water flow. Let us consider cooling a solid metal cylinder borrowing the initial data from $R_m = 0.01 \text{ m}_3$, $R_l = 0.013 \text{ m}$, $L = 0.2 \text{ m}$, $T_0 = 20 \text{ }^\circ\text{C}$. Let us take 0.1 ar m , the air initial temperature $T_0 = 0.1$, the initial temperature of the cylinder $950 \text{ Tm }^\circ\text{C}$, the saturation temperature $100 \text{ Ts }^\circ\text{C}$, and the water flow rate $u_0 = 12 \text{ m/s}$. The calculation time is 10 s . The thermophysical parameters of the media are taken according to





The temperature of the cylinder surface at the calculation points: 1, 2 – the temperatures calculated with the use of the offered model; 1', 2' – the temperatures calculated using the model described



The temperature in the middle of the cylinder surface when the presence of vapor $Y>0$ is taken into account, and when the presence of vapor $Y=0$ is not taken into account

When all the other conditions are equal, the higher the water flow rate is, the lower the received values of the temperature at the calculation points are. This regularity is more obvious in the initial region of the cooled surface. At the calculation point 1 within the time 5 s, the difference in the temperatures makes up 45% at the rates 1 m/s and 10 m/s. At the calculation point 2, when the rate of the flow decreases from 10 m/s to 1 m/s, the temperature increases by 4%, i.e. it remains practically the same for all the considered values of the water flow rate

Most instances were solved within a few minutes. Since the test problems are algebraically and computationally simple and small, the total time required for function evaluations for all runs was negligible. A limit of 10 CPU minutes was imposed on each run. As seen in this figure, no solver reached this CPU time limit for problems with up to nine variables. For problems with ten to thirty variables, only SID-PSM and SNOBFIT had to be terminated because of the time limit.

CONCLUSION

While this study was conducted on a small scale and its findings cannot be generalized, the utilization of the IGD framework and the APOS theory offered valuable insights into the teaching and learning processes surrounding the concepts of maximizing/minimizing in Grade 12 Mathematics classes. The examination revealed that certain aspects of the APOS theory were not fully operational, particularly as learners demonstrated proficiency mainly in tasks involving formula application and substitution. These manipulations often occurred externally, with learners relying on memorized rules and instructions. Within the APOS framework, some learners, such as those in Group 3, demonstrated progress toward the process stage, internalizing the volume formula and understanding the conditions for maxima/minima



occurrence. Notably, learner L2 appeared to have developed a schema for maxima/minima.

However, the study highlighted a partial alignment with the IGD framework, indicating a deficiency in schema development among learners attempting to construct knowledge of the concept. Instead, learners relied heavily on isolated facts and procedural understanding. This may reflect a teaching approach that prioritizes procedural aspects over conceptual comprehension, leading to a reliance on rote learning rather than deep understanding. The implications of these findings extend beyond the classroom setting. The mathematical modeling results derived from this study could be applied in various fields, including metallurgy and mechanical engineering. For instance, they could inform calculations regarding initial heat and hydrodynamic conditions, aiding in the cooling process of high-temperature cylindrical metal workpieces based on their geometry and thermophysical properties, as well as the characteristics of the cooling media and the duration of the process.

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