International Journal of Basic and Applied Research ISSN - 2249-3352
Vol. 02 Issue-2 (December 2012) (56-62)

# A Note on Pachpatte's Inequalities

**B. R. Sontakke [1], J.A. Nanware [2]**

[1] *Pratishthan Mahavidyalaya, Paithan, Dist. Aurangabad, INDIA.*
[2] *Shirkrishna Mahavidyalaya, Gunjoti, Dist. Osmanabad, INDIA.*

**E-mail: brsontakke@rediffmail.com, jag_skmg91@rediffmail.com**

**Key-words:**
Pachpatte's inequality,
Existence & uniqueness,
Integro-Differential equations.

**Abstract:** Our aim is to express the mathematical journey of Professor B.G. Pachpatte in brief. We also highlight the results investigated by him and its applications in solving many important problems.

## 1. INTRODUCTION AND BRIEF HISTORY :

Prof. B.G. Pachpatte was born on 21st Nov. 1943 in a small village Mudgad (Achoji) situated near Killari, earthquake prone area, Tq. Nilanga in Latur district of Marathwada region, where primary education was not available during those days. Pachpatte had to walk every day for school to another village Kokalgaon for five years. He studied from eight standard to B.Sc second year at another small town Omerga. He completed B.Sc honours and M.Sc in Mathematics at Aurangabad in 1966. He had a first class through out. After completion of M.Sc he immediately joined as a lecturer at Deogiri College, Aurangabad till 1977. He received Ph.D. in 1972 under the able guidance of Prof. S.G.Deo from Marathwada University. He joined the University Department in 1977 as Professor. Because of his studious nature, hard working capacity and keen interest in research, 15 students received Ph.D. under his guidance. He was a visiting Professor in the University of Texas at Arlington, U.S.A. under Indo-American fellowship during the year 1980-81. He retired from service in 2003 as Professor in Mathematics. He was a U.G.C. National Professor for one year and the Govt. of Maharashtra honored him as an Ideal University Teacher in 2003. He had published more than 577 papers in the reputed journals. He established his identity as Mathematician in USA and as honor he has taken on the editorial boards of internationally renowned journals such as Journal of Mathematical Analysis and Applications, Journal of Nonlinear Analysis : Theory, Methods and Applications, Octogan Mathematical Magazine (Romania), Australian Journal of Mathematical Analysis and Applications, Journal of Inequalities in Pure and Applied Mathematics (Australia), Mathematical Inequalities and Applications (Croatia), Differential Equations & Dynamical Systems (India), World Mathematical Reviews (Korea). His fields of interest includes: Mathematical Analysis and Applications, Analytic Inequalities, Ordinary and Partial Differential Equations, Integral Equations and Inequalities, Finite Difference Inequalities and Stochastic Analysis and Applications etc. He published five monographs which are:

1. Inequalities for Differential and Integral Equations, Academic press, New York, 1998.
2. Inequalities for finite Difference Equations, Marcel Dekker, Inc., New York, 2002.
3. Mathematical Inequalities, North-Holland Mathematical Library, Vol. 67, Elsevier Science, B.V., Amsterdam, 2005.
4. Integral and Finite Difference Inequalities and Applications, North-Holland Mathematics Studies, Vol. 2005, Elsevier

**International Journal of Basic and Applied Research**
**Vol. 02 Issue-2 (December 2012) (56-62)**

Science B.V. Amsterdam, 2006.

5. Multidimensional, Integral Equations and Inequalities, Atlantis Studies in Mathematics for Engineer and Science–2011, Volume 9.
6. Advances in Some Analytic Inequalities (in press).

Most of the results investigated by him are fundamental in respective fields. Many results given by him are used as tools by various researchers all over the world. Many of the ideas involved in his work are new and will encourage further research and widen the scope of their applications. Many basic results are now personalized by his name, to mention a few: Pachpatte's inequalities, Hilbert-Pachpatte type inequalities, Pachpatte-Poincare inequalities, Wendroff-Pachpatte inequalities, Pachpatte's Analytic inequalities, Pachpatte Ou-Lang inequalities etc.

## 2. LINEAR INTEGRAL INEQUALITY:

In 1973a, Pachpatte proved a very useful generalization of following well known Gronwall –Bellman inequality:

**Theorem 1** : Let $u$ and $f$ be continuous and nonnegative functions defined on $J=[\alpha,\beta]$ , and let c be a nonnegative constant. Then the inequality,

$$u(t) \le c + \int_\alpha^t f(s)u(s)ds, \quad t \in J$$ implies that

$$u(t) \le c \exp\left(\int_\alpha^t f(s)ds\right) \quad t \in J$$

**Theorem 2:** (Pachpatte's inequality) Let $u$, $f$ and $g$ be nonnegative continuous functions defined on $R_+$, for which the inequality

$$u(t) \le u_o + \int_o^t f(s)u(s)ds + \int_o^t f(s)\left(\int_o^s g(\sigma)u(\sigma)d\sigma\right)ds, t \in R_+$$

holds, where $u_o$ is a nonnegative constant. Then

$$u(t) \le u_o\left[1 + \int_o^t f(s)\exp\left(\int_0^s [f(s)+g(\sigma)]d\sigma\right)ds\right], t \in R_+$$

The above generalized inequality is applied in more general situations in which other available inequalities do not apply directly. The inequalities given by him in 1974a, 1975 b-d, 1977a are used to develop the theory of integral and integro-differential equations of more general type.

## 3.NONLINEAR INTEGRAL INEQUALITIES :

In order to study the qualitative properties of solutions of certain differential, integral and integro-differential equations some specific nonlinear integral inequalities play a vital role. The following fundamental inequality plays a vital role in the study of properties of solutions of certain differential, integral and integro-differential equations.

**THEOREM 3:** Let $u$, $f$ and $g$ be nonnegative continuous functions defined on $R+$, and $c>o$, $p>o$ and $p\neq 1$ are constants and suppose

$$u(t) \le c + \int_o^t f(s)u(s)ds + \int_o^t f(s)\left(\int_o^s g(\sigma)u^p(\sigma)d\sigma\right)ds, \quad t \in R_+$$

<i> if $o<p<1$ and $Eo(t)$ is defined by

$$E_o(t) = 1 + (1-p)c^{p-1}\int_o^t g(\tau)\exp\left(-(1-p)\int_o^\tau f(\sigma)d\sigma\right)d\tau,$$

then $u(t) \le c\left[1+\int_o^t f(s)\exp\left(\int_o^s f(\sigma)d\sigma\right)Eo(s)\right)^{1/(1-p)} ds\right]$, for $t \in R_+$

**<ii> if $1<p<\infty$, then**

$$u(t) \le c\left[1+\int_o^t f(s)\exp\left(\int_o^s f(\sigma)d\sigma\right)\left(\bar{E}o(s)\right)^{-1/(p-1)} ds\right], \quad ..(*)$$

for $t\in(o,\alpha)$, where $E^{-}o(t)$ is defined by the right-hand side of the definition of $Eo(t)$ for $1<p<\infty$ and $\alpha=\sup\{t\in R+ : E^{-}o(t)>o\}$. Moreover, if we assume that $E^{-}o(t)>o$ for all $t\in R_+$, the inequality (*) remains valid for all $t\in R_+$.

Pachpatte further generalized this inequality in 1976b, 1977 a, c, d and 1978 b [ 1].

Moreover Bihari like integral inequalities was established by him in 1974 d for the study of certain classes of general and integro-differential equations which is given below in more general form.

**THEOREM 3 :** Let $u$ and $g$ be nonnegative continuous functions defined on $R_+$. Let $H(u)$ be a continuous nondecreasing function defined on $R_+$ and $H(u)>o$ on $(o,\infty)$.

If

$$u(t) \le c + \int_o^t g(s)\left(u(s) + \int_o^s g(\sigma) H(u(\sigma)) d\sigma\right) ds,$$

for $t \in R+$, where $c \ge 0$ is a constant, then for $0 \le t \le t_1$,

$$u(t) \le c + \int_o^t g(s) G^{-1}\left[G(c) + \int_o^s g(\sigma) d\sigma\right] ds$$

where

$$G(r) = \int_{r_0}^{r} \frac{ds}{s + H(s)}, \quad r > 0, r_0 > 0,$$

and $G^{-1}$ is the inverse of G, and $t_1 \in R_+$ is chosen so that

$$G(c) + \int_0^t g(\sigma) d\sigma \in Dom(G^{-1}),$$

for all $t \in R_+$ lying in the interval $0 \le t \le t_1$.

The variants of above result are also obtained by him in 1976c.

Dragomir obtained the following inequality in 1987a,b, which is the generalization of well known Gronwall-Bellman inequality to study important properties of solution of certain differential equations.

**THEOREM 4 :** Let u, a, b : I =[α,β]→ $R_+$ be continuous functions. Let L:I × $R_+$ → $R_+$ be a continuous function such that

$$0 \le L(t,x) - L(t,y) \le M(t,y)(x-y)$$

for $t \in I$ and $x \ge y \ge 0$, where M:I×$R_+$→$R_+$ be continuous function. If

$$u(t) \le a(t) + b(t)\int_\alpha^t L(s, u(s)) ds \quad \text{for } t \in I, \text{ then}$$

$$u(t) \le a(t) + b(t)\int_\alpha^t L(s, a(s)) \exp\left(\int_s^t M(\sigma, a(\sigma)) b(\sigma) d\sigma\right) ds$$

for $t \in I$

Above inequality was generalized byPachpatte popularly known as Pachpatte's Inequality– I which is stated below:

**THEOREM 5 :** Let *u, a* and *b* be nonnegative continuous functions defined on $R_+$. Let *F(u)* be a continuous strictly increasing convex and submultiplicative function for $u > 0$, $\lim_{u \to \infty} F(u) = \infty$, $F^{-1}$ denotes the inverse function of F, $\alpha(t)$, $\beta(t)$ be continuous and positive functions defined on R+, $\alpha(t)+\beta(t)=1$, and L, M be the functions defined as L:J×$R_+$ → $R_+$ be continuous functions such that

$$0 \le L(t,x) - L(t,y) \le M(t,y)(x-y) \ldots\ldots(*)$$

for $t \in I$ and $x \ge y \ge 0$, where M:I×$R_+$→$R_+$ be a continuous function
satisfying the condition (*). If

$$u(t) \le a(t) + b(t) F^{-1}\left(\int_0^t L(s, F(u(s))) ds\right)$$

for $t \in R_+$, then

$$u(t) \le a(t) + b(t) F^{-1}\left(\int_0^t L(s, \alpha(s) F(a(s)\alpha^{-1}(s)))\right.$$

$$\left.\times \exp\left(\int_0^t M(\sigma, \alpha(\sigma) F(a(\sigma)\alpha^{-1}(\sigma))) \beta(\sigma) F(b(\sigma)\beta^{-1}(\sigma)) d\sigma\right) ds\right)$$

for $t \in R_+$

To study the qualitative properties of abstract differential and integral equations and class of partial differential equations some types of inequalities have played a crucial role such as Ou-Lang inequalities, generalized by him in 1995a, 1994a given as theorem 3.5.1, theorem 3.5.2, theorem 3.5.3, theorem 3.5.4, and theorem 3.5.5 in [1] popularly known as Pachpatte's Inequality- II. Pachpatte's Inequality-II was also more generalized by him in 1995 and popularly known as Pachpatte's inequality – III. The more general result is given below:

**THEOREM 6 :** Let *u, f, g* and *h* be nonnegative continuous functions defined on $R_+$ and *c* be a positive constant.

If

$$u^2(t) \leq c^2 + 2\int_a^t \left[ f(s)u^2(s) + g(s)u(s)\left( u(s) + \int_0^s h(\sigma)u(\sigma)d\sigma \right)\right] ds,$$

for $t \in R_+$, then

$$u(t) \leq c\Big[ \exp\left( \int_0^t f(s)ds \right) + \int_0^t g(s)\exp\left( \int_s^t f(\sigma)d\sigma \right)$$

$$\times \exp\left( \int_0^s [f(\sigma) + g(\sigma) + h(\sigma)]d\sigma \right) ds \Big],$$

for $t \in R_+$

The following comparison principle is popularly known as Pachpatte's Inequality – IV established by him in 1996a, 1996b [1] which is needed in the study of the analysis of differential, integral and integro-differential inequalities.

**THEOREM 7 :** Let u be a nonnegative continuous function defined on $R+$ and c be a nonnegative constant. Let $w(t,r)$ be a nonnegative continuous function defined for $t \in R+$, $0 \leq r < \infty$, and monotonic nondecreasing with respect to r for any fixed . If

$$u^2(t) \leq c^2 + 2\int_0^t u(s)w(s,u(s))ds,$$

for $t \in R+$

then $u(t) \leq r(t), t \in R_+$

where r(t) is the maximal solution of

$$r'(t) = w(t, r(t)),\ r(0) = c$$

For $t \in R+$

In 1996a, Professor Pachpatte established following integro-differential inequality:

**THEOREM 8 :** Let $u, u'$ and $a$ be nonnegative continuous functions defined on $I = [t_0, \infty], t_0 \geq 0$ with $u(t_0) = k$, a constant and c a nonnegative constant. If

$$(u'(t))^2 \leq c^2 + 2\int_{t_0}^t a(s)u'(s)(u(s) + u'(s))ds,$$

for $t \in I$, then

$$u(t) \leq k + \int_{t_0}^t \left[ c + (k+c)\int_{t_0}^s a(\tau)\exp\left( \int_{t_0}^t [1 + a(\sigma)]d\sigma \right) d\tau \right] ds,$$

for $t \in I$

Pachaptte's Inequality $V$, the outcome of generalization of Engler's inequality which is the slight variant of Haraux inequality given in theorem 3.8.2 [1] made by him in 1994d is stated below :

**THEOREM 9 :** Let $a : I \to R_+, u : I \to R_1$ are continuous functions and $u_0 \geq 1$ be a constant, where $I = [0, T]$ and $R_1 = [1, \infty)$ Let $f(u)$ be a continuous nondecreasing function defined on $R_+$ and $f(u) > 0$ for $u > 0$, If

$$u(t) \leq u_0 + \int_0^t a(s)u(s)f(\log u(s))ds$$

for $t \in I$

than for $0 \leq t \leq t_1$

$$u(t) \leq \exp\left( \bar{F}^1 \left[ F(\log u_0) + \int_0^t a(s)ds \right]\right)$$

where

$$F(r) = \int_{r_0}^r \frac{ds}{f(s)}, \quad r > 0, r_0 > 0$$

$F^{-1}$ is the inverse of F and $t_1 \in I$ is chosen so that

$$F(\log u_0) + \int_0^t a(s)ds \in Dom(\bar{F}^1)$$

for all $t \in I$ lying in interval $0 \leq t \leq t_1$.

## 4. MULTIDIMENSIONAL LINEAR INTEGRAL INEQUALITIES :

The dynamics of physical systems is governed by various classes of nonlinear hyperbolic partial differential equations.

To analyze the dynamical system multidimensional differential, integral and integro-differential inequalities play an important role in the analysis of nonlinear hyperbolic partial differential equations. In fact he had generalized inequalities in 1995 such as

Wendroff and Snow which are widly used in various applications. These results are given below:

**4.1 Pachpatte-Wendroff Inequality :**

Let $u(x,y)$, $p(x,y)$ and $q(x,y)$ be nonnegative continuous functions defined for $x,y \in R+$. Let $k(x,y,s,t)$ and its partial derivatives $k_x(x,y,s,t)$, $k_y(x,y,s,t)$, $k_{xy}(x,y,s,t)$ be nonnegative continuous functions for $0 \le s \le x < \infty, 0 \le t \le y < \infty$ If

$$u(x,y) \le p(x,y) + q(x,y)\int_0^x\int_0^y k(x,y,s,t)u(s,t)dsdt,$$

for $x,y \in R_+$ ,then

$$u(x,y) \le p(x,y) + q(x,y)\left(\int_0^x\int_0^y A(\sigma,\tau)d\sigma\, d\tau\right) \times \exp\left(\int_0^x\int_0^y B(\sigma,\tau)d\sigma\, d\tau\right)$$

for $x,y \in R_+$ , where

$$A(x,y) = k(x,y,x,y)p(x,y) + \int_0^x k_x(x,y,s,y)p(s,y)ds + \int_0^y k_y(x,y,s,t)p(x,t)dt + \int_0^x\int_0^y k_{xy}(x,y,s,t)p(s,t)dsdt,$$

$$B(x,y) = k(x,y,x,y)q(x,y) + \int_0^x k_x(x,y,s,y)q(s,y)ds + \int_0^y k_y(x,y,x,t)q(x,t)dt + \int_0^x\int_0^y k_{xy}(x,y,s,t)q(s,t)dsdt,$$

for $x,y \in R_+$

**4.2 Pachpatte-Snow's Inequality :**

**THEOREM 11 :** Suppose $u(x,y), a(x,y), b(x,y), c(x,y)$ and $\sigma(x,y)$are nonnegative continuous functions on a domain D. Let $P_0(x_0,y_0)$and $P(x,y)$be two points in D such that $(x-x_0)(y-y_0) \ge 0$ and let R be the rectangular region whose opposite corners are the points $P_0$and $P$. Let $v(s,t;x,y)$ be the solution of the characteristic initial value problem

$$L[v] = v_{st} - [b(s,t) + c(s,t)]v = 0$$

$$v(s,y) = v(x,t) = 1$$

and let $D^+$ be a connected subdomain of D which contains P and on which $v \ge 0$. If $R \subset D^+$and $u(x,y)$satisfies,

$$u(x,y) \le a(x,y) + \int_{x_0}^x\int_{y_0}^y b(s,t)u(s,t)dsdt + \int_{x_0}^x\int_{y_0}^y b(s,t)\left[\sigma(s,t) + \int_{x_0}^s\int_{y_0}^t c(\xi,\eta)u(\xi,\eta)d\xi d\eta\right] dsdt,$$

than $u(x,y)$ also satisfies

$$u(x,y) \le a(x,y) + \int_{x_0}^x\int_{y_0}^y b(s,t)\Big[a(s,t) + \sigma(s,t) + \int_{x_0}^s\int_{y_0}^t [a(\xi,\eta)c(\xi,\eta) + b(\xi,\eta)(a(\xi,\eta) + \sigma(\xi,\eta))] \times v(\xi,\eta;s,t)d\xi\, d\eta\Big]dsdt \quad \text{.....(A)}$$

where the function $v(s,t,x,y)$ involved in (A) is the Riemann function relative to the point $P(x,y)$ for self-adjoint operator L.

The multidimensional nonlinear integral inequalities are also established by Pachpatte and his research students.

**Note :** Analytic inequalities and finite difference inequalities have been developed by him. See details in [2,3,4,5,6].

**5. FRACTIONAL INEQUALITIES :**

We present here some of the well known inequalities generalized by Professor Pachpatte such as Hilbert-Pachpatte inequality, Hilbert-Pachpatte type inequalities, which are further generalized by Anasstassiou [8] to fractional integral inequalities.

**5.1 Hilbert-Pachpatte Integral Inequality :**

**THEOREM 12 :**

For $i \in \{1,...,n\}$, take $x_i > 0$ and assume

$u_i \in L_1(0,x_i)$, $g_i \in L_\infty((0,x_i)^2)$ , $\Phi_i \in L_\infty(0,x_i)$

with $g_i, \Phi_i > 0$ , take $r_i \ge 0$

$p_i, q_i > 1 : \frac{1}{p_i} + \frac{1}{q_i} = 1$ ,

and $w_i > 0$ such that $\sum_{i=1}^{n} w_i = \Omega n$

and $a_i, b_i \in [0,1]$ such that $a_i + b_i = 1$ . call

$$\varphi_i(s_i) := \int_0^{s_i} \left(g_i(s_i, \tau_i)\right)^{(a_i + b_i q_i)^{r_i}} d\tau_i$$

Suppose φ*i*(*si*)>0 with the exception φ*i*(0)=0. If

$$|u_i(s_i)| \le \int_0^{s_i} \left(g_i(s_i, \tau_i)\right)^{r_i} \Phi_i(\tau_i) d\tau_i$$

$s_i \in [0, x_i], i = 1, \ldots, n$ , than

$$I_1 := \int_0^{x_1} \cdots \int_0^{x_n} \frac{\prod_{i=1}^{n} |u_i(s_i)|}{\left(\frac{1}{\Omega_n} \sum_{i=1}^{n} w_i (\varphi_i(s_i))^{1/q_i w_i}\right)^{\Omega_n}} ds_1 \ldots ds_n$$

$$\le \prod_{i=1}^{n} x_i^{1/q_i} \left[ \int_0^{x_i} \phi_i(\tau_i)^{p_i} \left( \int_{\tau_i}^{x_i} g_i(s_i, \tau_i)^{a_i r_i} ds_i \right) d\tau_i \right]^{1/p_i}$$

**5.2 Hilbert-Pachpatte Type Inequality :**

**THEOREM 12 :** Let $v_i \in L_1(Q(x_i))$, $\Phi_i \in L_\infty(Q(x_i))$ $\Phi_i \ge 0$, $c_i > -1$, $c_i \in R^d$ , $i = 1, \ldots, N$

set $U = \frac{1}{\sum_{i=1}^{N} \left[ (\alpha_i + 1)^{1/q_i} (\beta_i + 1)^{1/q_i} \right]}$

Let $Ji := \{ j \in \{1, \ldots, d\} -1 < c_i^j < 0 \}$

if $Ji \ne \phi$ , we take

$$b_i > \max_{j \in J_i} \left( \frac{(1 + c_i^j)}{c_i^j (1 - q_i)} \right)$$

suppose $|V_i(s_i)| \le \int_0^{s_i} (s_i - \tau_i)^{c_i} \Phi(\tau_i) d\tau_i$

$\forall s_i \in Q(x_i)$ , $i = 1, \ldots, N.$ than

$$M := \int_0^{x_1} \cdots \int_0^{x_N} \frac{\prod_{i=1}^{N} |V_i(s_i)| ds_1 \ldots ds_N}{\left(\frac{1}{\Omega_N} \sum_{i=1}^{n} w_i s_i^{(\alpha_i + 1)/q_i w_i}\right)^{\Omega_N}}$$

$$\le U \prod_{i=1}^{N} x_i^{1/q_i} \prod_{i=1}^{N} \left[ \int_0^{x_i} (x_i - s_i)^{\beta_i + 1} \phi_i(s_i)^{p_i} ds_i \right]^{1/p_i}$$

The application of above result is given as theorem 20.25 in [8].

**5.3 Hilbert-Pachpatte Type Fractional Integral Inequalities :**

We present here very general weighted univariate and multivariate Hilbert-Pachpatte type integral inequalities involving Caputo and Riemann – Liouville fractional derivatives and fractional partial derivatives.

**THEOREM 13 :** Let $i \in \{1, \ldots, n\}$, $x_i \ge 0$, $\gamma_i \ge 0$, $v_i \ge \gamma_i + 1$.

Let $f_i \in AC^{n_i}([0(x_i)])$, $n_i = [v_i]$

such that $f_i^{(k_i)}(0) = 0, k_2 = 0, 1, \ldots, n_i - 1$

and $D_*^{v_i} f_i \in L_\infty(0, x_i)$

Let $p_i, q_i > 1 : \frac{1}{p_i} + \frac{1}{q_i} = 1$ , $w_i > 0$

such that $\sum_{i=1}^{n} w_i = \Omega_n$

and $a_i, b_i \in [0,1]$

such that $a_i + b_i = 1$ , than

$$I_1 := \int_0^{x_1} \cdots \int_0^{x_n} \frac{\prod_{i=1}^{n} |u_i(s_i)|}{\left(\frac{1}{\Omega_n} \sum_{i=1}^{n} w_i (\varphi_i(s_i))^{1/q_i w_i}\right)^{\Omega_n}} ds_1 \ldots ds_n$$

$$\le C \prod_{i=1}^{n} x^{1/q_i} \left[ \int_0^{x_i} \left| D_{*0}^{v_i} f_i(t_i) \right|^{p_i} (x_i - t_i)^{\{a_i(v_i - \gamma_i - 1) + 1\}} dt_i \right]^{1/p_i}$$

where

$$C : \prod_{i=1}^{n} \left( \frac{1}{\Gamma(v_i - \gamma_i)\left(a_i(v_i - \gamma_i - 1) + 1\right)^{1/p_i}} \right)$$

**THEOREM 14 :** Let

$$f:[0,x]\times[0,y]\rightarrow R, x, y\in R_{+}-\{0\}.$$

such that

$$f(\cdot,s_2)\in AC^n[0,x], \forall s_2\in[0,y]$$

Assume that

$$f\in C^{(n,m)}([0,x]\times[0,y])$$

then

$$D_{*s_1s_2}^{v+u}\ f(s_1,s_2)=D_{*s_2s_1}^{u+v}f(s_1,s_2)$$

for all

$$(s_1,s_2)\in[0,x]\times[0,y]$$

**AKNOWEDGEMENT:**

Authors are thankful to Prof. D. B. Dhaigude for his encouragement while preparing the article.

**REFERENCES :**

(1) B.G.Pachpatte : Inequalities for Differential and Integral Equations, Academic Press, New York,1988.

(2) B.G.Pachpatte : Inequalities for Finite Difference Equations, Marcel Dekker, New York, 2002.

(3) B.G.Pachpatte : New Chebysev Type Inequalities Via Tripezoidal Like Rules, Journal of Inequalities in Pure and Applied Mathematics, Vol.7, issue 1, Art 31, 2006.

(4) B.G.Pachpatte : On Chebysev Type Inequalities Involving Functions Whose Derivatives Belongs to Lp Spaces, Journal of Inequalities in Pure and Applied Mathematics, Vol.2, Art 58, 2006.

(5) B.G.Pachpatte : New Oslrowski Type Inequalities Via Mean Value Theorems, Journal of Inequalities in Pure and Applied Mathematics, Vol.7, issue 4, Art 137, 2006.

(6) B.G.Pachpatte : A Note on Integral Inequalities Involving Product of Two Functions, Journal of Inequalities in Pure and Applied Mathematics, Vol.7, issue 2, Art 78, 2006.

(7) Bulletin of the Marathwada Mathematical Society, Vol.4, 2003.

(8) G.A. Anasstassiou : Fractional Differentiation Inequalities, Springer, New York, 2008.